Chapter 4—Outline

Visual Instruments

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Chapter 4

Visual Instruments

In this chapter, we study the properties of simple magnifiers and microscopes, instruments that aid the viewing of nearby objects; we also study telescopes, instruments used to view distant objects. All the instruments are located in air and the lenses are treated as thin. We use the methods of paraxial optics that are described in Chapter 1.

4.1 Magnifying Nearby Objects

4.1.1 Introduction

The common purpose of a simple magnifier or microscope is to make an object viewed by the eye appear larger and with better resolution than when the object is viewed by the unaided eye. Usually, the unaided eye sees an object best when it is brought as close-as-possible to the eye, called the near point; this close-as-possible distance for the normal eye is 250 mm on the average; for young people this distance is smaller, for older, it is larger. We diagram this situation in Figure 4.1(a), where we show a simple model for the human eye. We represent the distance to the near point by the symbol d_0, and because we draw the dimension arrow pointing to the left, we write $d_0 = -250$ mm, in accord with our sign convention which follows the rules of analytic geometry (the δs are the angles of inclination and are also negative). As usual, the z axis is the symmetry axis.

The eye is almost spherical with a diameter of approximately 23 mm, and is filled with a jelly-like fluid with index of refraction $n'_{\text{eye}} = 1.337$. The eyelens is a layered, plastic-like, crystalline lens with an index of refraction of about 1.40. Because of its layered structure, the eyelens is flexible; muscles surrounding it change the curvature of its surfaces to provide more bending power (shorter focal length) so that nearby objects are seen, or less bending power (longer focal length) so that distant objects are seen—a process called accommodation. Although the images formed on the retina are inverted; the brain and eye are "wired" such that we see the images as erect. The model for the eye that we have given is greatly simplified, but it will serve to illustrate the properties of the visual instruments we shall investigate.

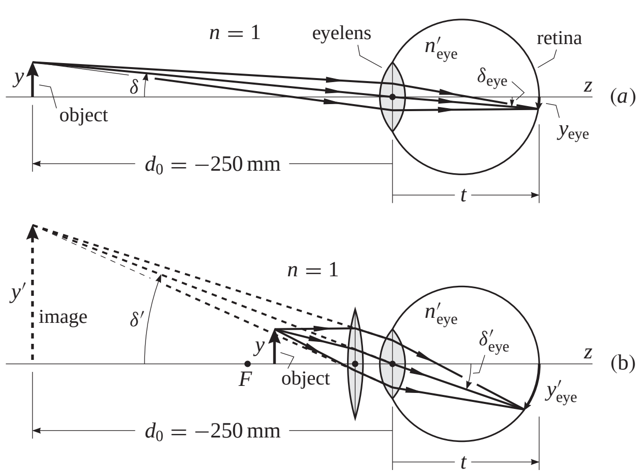

Figure 4.1 The simple magnifier: both the eyelens and the positive lens are treated as thin in these ray diagrams.

To illustrate how we attack this problem, we use the simple magnifier illustrated in Figure 4.1(b), a single positive lens (also called a convex or converging lens). When we scan through the diagrams of Figure 1.80, we see that the arrangement in Figure 1.80(e) provides a convenient way of making an enlarged image (we assume the eye is positioned to the right in those diagrams). In fact, placing the object anywhere between the lens and the unprimed focal point F produces an enlarged image. It is true that the image formed is virtual, but as the ray diagram shows in Figure 4.1(b), the eye has no trouble seeing such an image. Figure 1.80(d) indicates that when the object is at F the image is formed at infinity, moving the object to the right of F moves the image to the right of $-\infty$: remember, as we discussed at the end of Section 1.4.6, an image always moves in the same direction as the object. The image moves much more rapidly than the object does, and eventually reaches the near point, the situation shown in Figure 4.1(b), where we see the image is larger than the one in (a), even though the eye sees the image the same distance d_0 away. With the use of the positive lens in Figure 4.1(b), we are able to bring the object closer to the eye than the 250 mm in (a).

We now define the magnifying power MP by comparing the image heights on the retina in Figures 4.1(a) and (b):

$$\text{MP} = \frac{y'_{\text{eye}}}{y_{\text{eye}}} \tag{4.1}$$

However, it is difficult to measure these heights, so a more useful formula is desirable. Referring to Equation 1.105 and Figure 1.53, we have

$$n\,\delta = n'\,\delta' \tag{4.2}$$

and diagram the meaning for a thin lens in Figure 4.2 (the principal points H, H' coincide at the center of a thin lens).

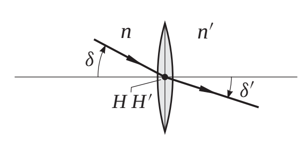

Figure 4.2

To make it easier to use Equation 4.2 on the diagrams in Figure 4.1, we redraw them in Figure 4.3(a) and (b). Remembering that $n = 1$, we apply Equation 4.2 to the ray that passes through the center of the eyelens in the diagrams of Figure 4.3(a) and (b), and obtain

$$\delta = n'_{\text{eye}}\,\delta_{\text{eye}} \qquad \delta' = n'_{\text{eye}}\,\delta'_{\text{eye}} \tag{4.3}$$

For the small δ angles in these same diagrams, we see that

$$y_{\text{eye}} = t\,\delta_{\text{eye}} \qquad y'_{\text{eye}} = t\,\delta'_{\text{eye}} \tag{4.4}$$

Using Equations 4.3 and 4.4 in Equation 4.1, we obtain a more useful equation for the magnifying power MP:

$$\text{MP} = \frac{y'_{\text{eye}}}{y_{\text{eye}}} = \frac{t\,\delta'_{\text{eye}}}{t\,\delta_{\text{eye}}} = \frac{\delta'/n'_{\text{eye}}}{\delta/n'_{\text{eye}}} = \frac{\delta'}{\delta} \tag{4.5}$$

This equation is the basic one for determining MP: It is a comparison between what the aided and unaided eye sees, and is calculated by the ratio of the angles δ' and δ subtended at the eye.

The image the aided eye sees is not necessarily at the near point. Because the eye accommodates, the image can be located anywhere between the near point and $-\infty$. As indicated by the diagrams in Figure 4.3(b) and (c), this image shift is easily accomplished by moving the object towards the F point; when the object is at F the image is at $-\infty$. Often, the image is placed at $-\infty$, simply because it simplifies equations. It was also thought that the eye muscles were most relaxed when viewing an image at $-\infty$, but recent research indicates that a distance of approximately a meter is probably more favorable.

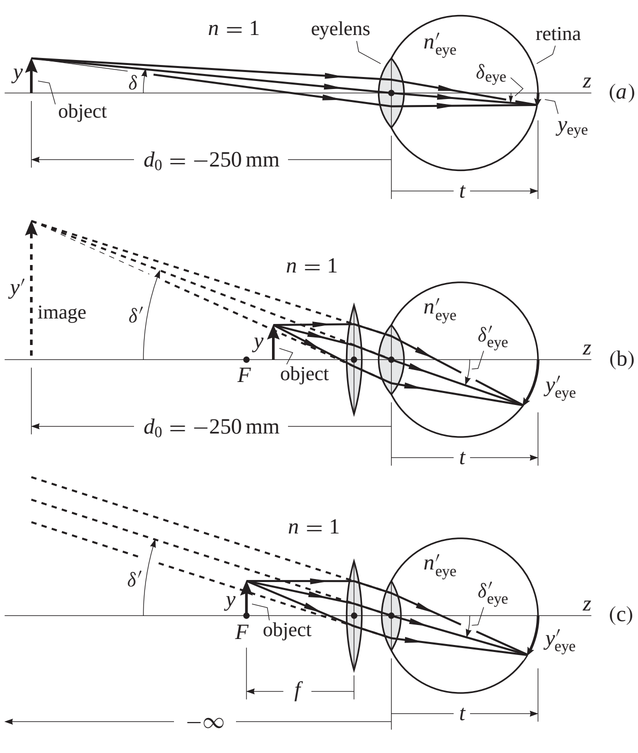

Figure 4.3

4.1.2 Theory for a general lens system

In Figure 4.4(a), the unaided eye views an object of height y at the near point, whereas in (b) the same object is viewed by the aided eye as the virtual image of height y'. The virtual image is produced by a generalized lens system shown as a converging (or positive) system for convenience; however, it can be any kind of system, and the object does not have to be to the right of F, it can be to the left so that a real image is formed. This lens system represents not only a simple magnifier, but also more complicated magnifiers, such as the compound microscope. We now want to derive several important equations for this generalized system.

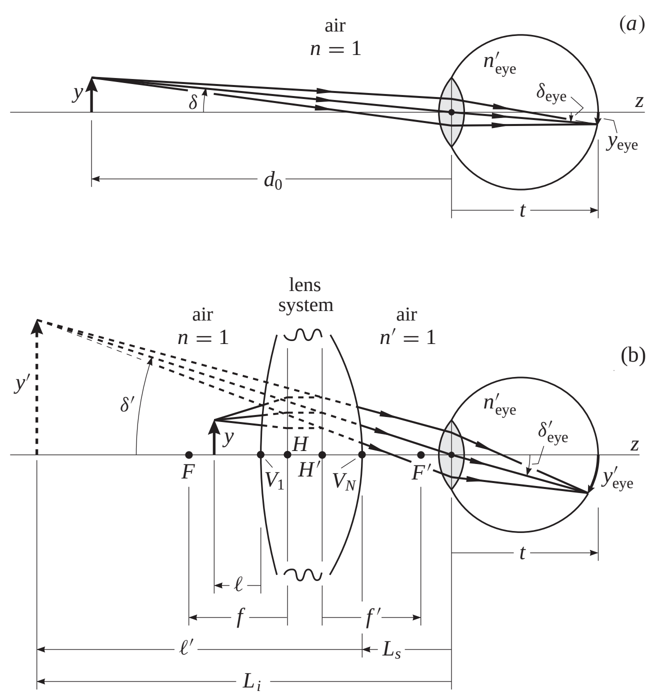

Figure 4.4

First of all, we see that when the method for deriving Equation 4.5 is applied to the diagrams in Figure 4.4, we get the same equation, namely

$$\text{MP} = \frac{\delta'}{\delta} \tag{4.6}$$

Next, we look at (b) to obtain y'/L_i for δ', and at (a) to obtain y/d_0 for δ; therefore

$$\text{MP} = \frac{\delta'}{\delta} = \frac{y'/L_i}{y/d_0} = \frac{y'}{y}\frac{d_0}{L_i} = m_T\,\frac{d_0}{L_i} \tag{4.7}$$

where, by Equation 1.75, $m_T = y'/y$ is the transverse or lateral magnification. We see that there is a close relationship between MP and m_T: they differ by the factor d_0/L_i. It is useful to remember that the d_0 in Equation 4.7 represents the distance to the near point in Figure 4.4(a) where the unaided eye views the object. We now look at several cases.

Case 1) $L_i = d_0$**.** In this case, we imagine the image moves to the near point position in Figure 4.4(b). Setting $L_i = d_0$ in Equation 4.7, we obtain

$$\text{MP} = m_T \tag{4.8}$$

Thus, we see that MP and m_T are the same when $L_i = d_0$. We can use Equations 1.77 and 1.76 to write m_T in terms of the Gaussian constants with $n = n' = 1$:

$$m_T = \frac{1}{b + a\ell} = \frac{1}{b + \ell/f'} \tag{4.9a}$$

$$m_T = c - a\ell' = c - \ell'/f' \tag{4.9b}$$

where by Equation 1.129b, $a = n'/f' = 1/f'$.

Case 2) A general form for MP**.** It is often more convenient to express MP in terms of ℓ' rather that ℓ; thus, substituting Equation 4.9b into Equation 4.7, we have

$$\text{MP} = \left(c - \frac{\ell'}{f'}\right)\frac{d_0}{L_i} = \left(\frac{c}{\ell'} - \frac{1}{f'}\right)\frac{d_0}{1 + L_s/\ell'} \tag{4.10}$$

where in the last step, we used $L_i = \ell' + L_s$ (see Figure 4.4) and divided through by ℓ'. When we want to know ℓ, we solve the ℓ, ℓ' relation of Equation 1.73

$$\ell' = \frac{d + c\ell}{b + a\ell} = \frac{d + c\ell}{b + \ell/f'} \tag{4.11a}$$

for ℓ to get

$$\ell = \frac{d - b\ell'}{-c + a\ell'} = \frac{d - b\ell'}{-c + \ell'/f'} \tag{4.11b}$$

Remember $n = n' = 1$ for all our work on visual instruments.

Case 3) $\ell' \to -\infty$**.** When the object moves to the F position, the image recedes to $-\infty$; that is, looking at Figure 4.4(b), both $\ell' \to -\infty$ and $L_i \to -\infty$. Allowing $\ell' \to -\infty$ in Equation 4.10, we quickly see that the expression for MP simplifies to

$$\text{MP} = \frac{-d_0}{f'} = \frac{250\,\text{mm}}{f'} \tag{4.12}$$

where we have replaced the distance d_0 to the near point by its normal eye value of -250 mm. We mentioned at the end of Section 4.1.1 that placing the image at $-\infty$ produced simple equations, and we see an example of this behavior in this equation for MP. We also see that MP increases with a decrease in f'.

4.1.3 The simple magnifier

We have already worked with the simple magnifier (also called a simple microscope or reading glass) in Section 4.1.1. But with the help of the previous section on a general lens system, we now have additional information on its properties. In the notation of Figure 4.4(b), we draw the diagram in Figure 4.5 for a simple magnifier with the object at ℓ and the virtual image at ℓ'.

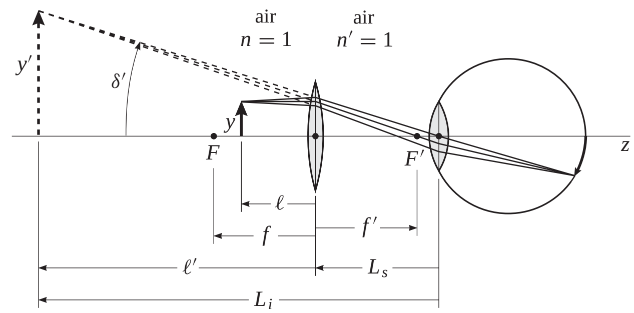

Figure 4.5 An erect, virtual image is produced.

The Gaussian constants for a thin lens are given by Equations 1.160 as

$$a = 1/f' \qquad b = 1 \qquad c = 1 \qquad d = 0 \tag{4.13}$$

We use these constants in the examples that follow.

Example 4.1.1 A simple magnifier with $L_s = 0$**.** Suppose a thin converging lens of 50.0 mm focal length is used as a simple magnifier. If the lens is held to form a virtual image 250 mm from the lens and $L_s = 0$, we want to determine the magnifying power MP and the distance ℓ of the object from the lens. In our terminology, $f' = 50.0$ mm, $L_i = \ell' = -250$ mm, and $d_0 = -250$ mm. With this information and Equation 4.13, we use Equation 4.10 to obtain

$$\begin{aligned}\text{MP} &= \left(\frac{c}{\ell'} - \frac{1}{f'}\right)\frac{d_0}{1 + L_s/\ell'} \\ &= \left(\frac{1}{-250} - \frac{1}{50}\right)\frac{-250}{1} = 6\end{aligned} \tag{4.14}$$

and Equation 4.11b to get

$$\ell = \frac{d - b\ell'}{-c + \ell'/f'} = \frac{0 - (1)(-250)}{-1 + (-250)/50} = -41.7\,\text{mm}$$

To place the image at $\ell' = -\infty$ mm, the object must move to the F position, which means that ℓ goes from -41.7 mm to -50.0 mm, a distance of only -8.3 mm. By Equation 4.12, the corresponding magnifying power is

$$\text{MP} = \frac{-d_0}{f'} = \frac{250\,\text{mm}}{50} = 5$$

a change from MP $= 6$ of 17 %—not much when we consider the change in the distance of ℓ', from -250 mm to $-\infty$.

To see in more detail how MP varies with ℓ', we write the expression in Equation 4.14 in terms of ℓ' as

$$\text{MP} = \left(\frac{1}{\ell'} - \frac{1}{50}\right)\frac{-250}{1} = \left(\frac{-250}{\ell'} + 5\right)$$

and then draw the graph in Figure 4.6. We observe most of the variation in MP occurs soon after the image moves to the left of -250 mm. We use Equation 4.11b to calculate that ℓ changes from -41.7 to -49.8 mm when ℓ' changes from -250 to $-10\,000$ mm, again we see that ℓ changes by a small amount while ℓ' changes by a large one.

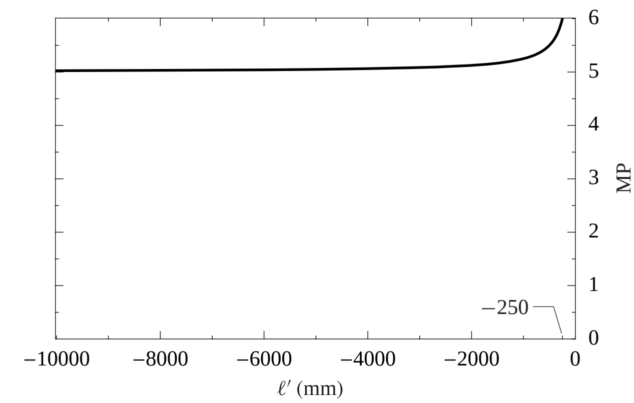

Figure 4.6

Example 4.1.2 A reading glass.
An elderly person for whom $d_0 = -400$ mm looks at a page of text. Continuing to hold the page at the d_0 distance from the eyes, the person places a reading glass of $f' = 240$ mm on the page, and then slowly moves it toward the eyes. We want to investigate the behavior of MP.

In the context of Figure 4.5, we see that

$$L_o = \ell + L_s = d_0 = -400\,\text{mm} \tag{4.15}$$

where we use L_o to represent the constant distance between the page (the object) and the eyes. Because ℓ is important in this example, we replace ℓ' in Equation 4.10 with Equation 4.11a, use Equation 4.13 for the Gaussian constants, and substitute L_s for $L_o - \ell$ from Equation 4.15. We obtain after some rearrangement the equation for MP:

$$\text{MP} = \frac{d_0 f'}{L_o f' + L_o \ell - \ell^2} \tag{4.16}$$

Substituting the given values, we get

$$\text{MP} = \frac{96000}{96000 + 400\,\ell + \ell^2} \tag{4.17}$$

We graph the magnifying power MP of Equation 4.17 from $\ell = 0$ to $\ell = -200$ mm in Figure 4.7. We see that MP gradually increases as ℓ becomes more negative; that is, as the person moves the reading glass away from the page.

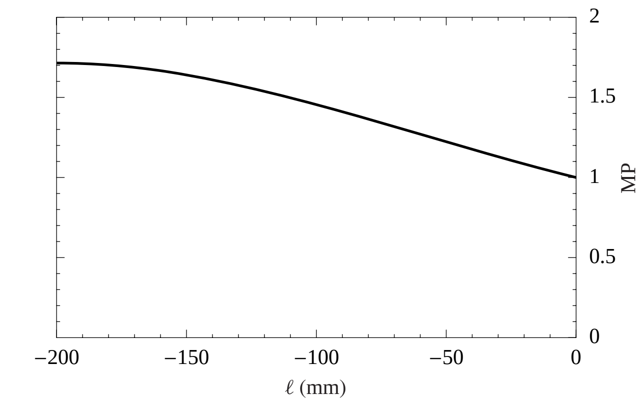

Figure 4.7

Example 4.1.3 A simple magnifier and a real image.
A thin converging lens of focal length 50 mm is positioned 75 mm from some print on a page. The eye of the observer is 475 mm from the page; the distance to the observer's near point is 250 mm. We want to investigate this situation.

We draw a diagram to represent this example in Figure 4.8. Symbolically, we have $f' = 50$ mm, $\ell = -75$ mm, $L_o = -475$ mm, and $d_0 = -250$ mm.

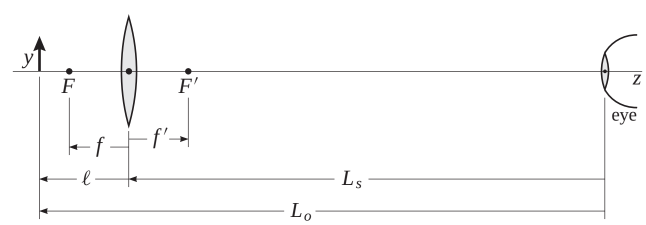

Figure 4.8

The information given indicates that the equation we need to calculate MP is Equation 4.16:

$$\begin{aligned}\text{MP} &= \frac{d_0 f'}{L_o f' + L_o \ell - \ell^2} \\ &= \frac{(-250)(50)}{(-475)(50) + (-475)(-75) - (-75)^2} = -2\end{aligned}$$

Recalling Equation 4.13, which says that $b = c = 1$, $d = 0$, we use Equation 4.11a to get

$$\ell' = \frac{d + c\ell}{b + \ell/f'} = \frac{0 + (1)(-75)}{1 + (-75)/(50)} = 150\,\text{mm}$$

So far, we see that MP $= -2$ means that the image the eye sees is twice the size of the object, and inverted. The positive value of ℓ' signifies that the image is real.

In Figure 4.9, we draw a diagram similar to Figure 4.8, but expand it to include the results we have obtained so far. By inspection of this diagram, we obtain the distance of the lens from the eye as

$$L_s = L_o - \ell = -475 - (-75) = -400\,\text{mm}$$

and the distance of the image from the eye is

$$L_i = L_s + \ell' = -400 + 150 = -250\,\text{mm}$$

Remember, to obtain these relations, treat the dimension arrows as vectors: then add or subtract them like vectors. From the value of L_i, we see that the observer views the image at the near point. According to Case 1) in Section 4.1.2 when $L_i = d_0$, then Equation 4.8 gives $\text{MP} = m_T$, and we can calculate m_T with either Equation 4.9a or 4.9b. Choosing the latter equation, we have

$$\text{MP} = m_T = c - \ell'/f' = 1 - 150/50 = -2$$

in agreement with the value of MP that we calculated earlier.

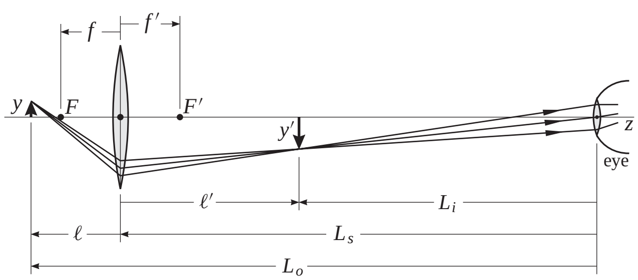

Figure 4.9

Example 4.1.4 A simple magnifier for the farsighted. A farsighted person with a distance of 600 mm to the near point uses a thin converging lens of focal length $f' = 120$ mm to view an object. The person holds the lens close enough to the eye so that $L_s = 0$ and views the virtual image at the near point. We want to calculate the magnifying power MP and the distance ℓ of the object from the lens.

Along with the quantities already given, we write the image distance $\ell' = -600$ mm, and $d_0 = -600$ mm. For a thin lens, $b = c = 1$, and $d = 0$. Substituting these values into Equation 4.10, we calculate

$$\begin{aligned}\text{MP} &= \left(\frac{c}{\ell'} - \frac{1}{f'}\right)\frac{d_0}{1 + L_s/\ell'}\\ &= \left(\frac{1}{-600} - \frac{1}{120}\right)\frac{-600}{1+0} = 6\end{aligned}$$

With Equation 4.11b, we get

$$\ell = \frac{d - b\ell'}{-c + \ell'/f'} = \frac{0 - 1(-600)}{-1 + (-600)/120} = -100\,\text{mm}$$

We diagram this optical system in Figure 4.10. We do not show f' and F' in the diagram for lack of space.

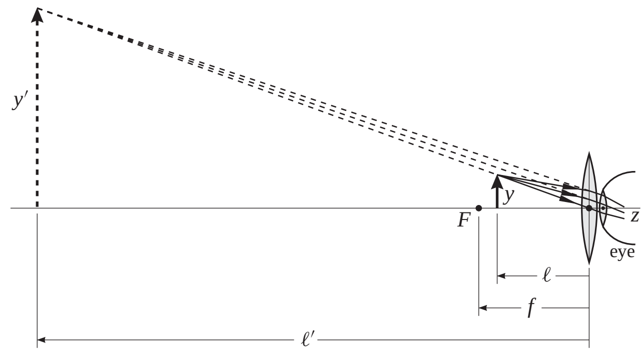

Figure 4.10

Example 4.1.5 A simple magnifier and the L_s effect. In this example, we want to investigate the effect of changing L_s for the normal eye with $d_0 = -250$ mm. The meaning of L_s and the other quantities of importance are displayed in Figure 4.11. The given values are $f' = 60$ mm, $\ell = -48$ mm, and $L_s = 0, -10, -20$ mm. Again, for lack of space, we do not show f' or F' in the diagram, but we should imagine that they are to the right of the lens. With the usual thin-lens values for b, c, d and Equation 4.11a, we get

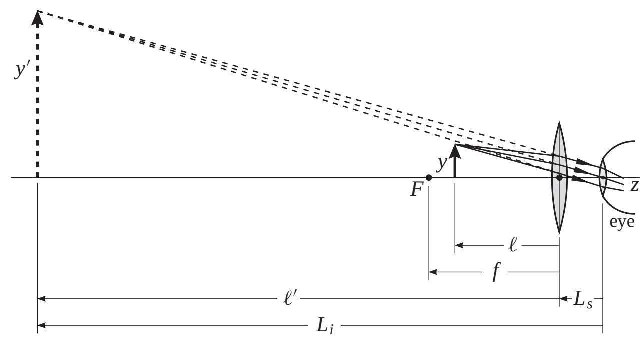

Figure 4.11

$$\ell' = \frac{d + c\ell}{b + \ell/f'} = \frac{0 + (1)(-48)}{1 + (-48)/60} = -240\,\text{mm}$$

and using Equation 4.10, we calculate

$$\text{MP} = \left(\frac{c}{\ell'} - \frac{1}{f'}\right)\frac{d_0}{1 + L_s/\ell'} = 5.21,\ 5,\ 4.81$$

after substituting the given values, as well as the successive values of L_s. It is also important to calculate L_i because it gives the distance from eye to image:

$$\begin{aligned}L_i &= L_s + \ell' = (0,\ -10,\ -20) - 240\\ &= -240,\ -250,\ -260\,\text{mm}\end{aligned}$$

Because $d_0 = -250$ mm, the image for $L_i = -240$ mm is not seen distinctly, the others are.

4.1.4 The compound microscope

Simple magnifiers produce magnifying powers MP that are rather small, approximately 2 to 20; aberrations limit the size of the MP for a simple magnifier. For larger magnifying powers, two lenses are used to form a compound microscope; this device allows the examination of very small objects. A thin-lens version of a compound microscope in air is shown in Figure 4.12. In this diagram, the objective lens is a converging lens that gathers the rays from the object of height y and focuses them to form a real, inverted (and larger) image of height y_1'. The eyepiece is another converging lens that acts as a simple magnifier of the real, inverted image to form a virtual image (still inverted and larger yet) of height y'. In Figure 4.12, we show both the unprimed and the primed focal points (F_o, F_o') of the objective, but only the unprimed focal point F_e of the eyepiece—the primed focal point F_e', and its corresponding focal length f_e', are not shown for lack of space. The direction of the dimension arrows indicate that for the objective $f_o' > 0$, $f_o < 0$, and because the lens is in air, $f_o' = -f_o$. Similarly, for the eyepiece, $f_e' > 0$, $f_e < 0$ with the relationship $f_e' = -f_e$. For simplicity, we use just two rays of the three-ray method we described in Example 1.5.1 and Figure 1.78 to locate the images in Figure 4.12.

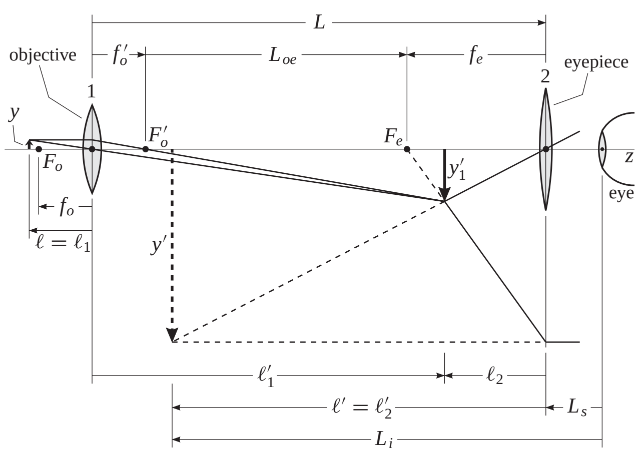

Figure 4.12 The compound microscope is made of two lenses: the objective and the eyepiece (or eyelens).

The distance L between the lenses in Figure 4.12 is called the mechanical tube length, and the distance L_{oe} is named the optical tube length; we normally use L_{oe} rather than L to obtain somewhat simpler equations. By inspection of Figure 4.12, and with $f_e' = -f_e$, we see that

$$L = f_o' + L_{oe} - f_e = f_o' + L_{oe} + f_e' \tag{4.18}$$

To obtain the simplest relationship for the magnifying power MP for the compound microscope, we move the final image y' to $-\infty$; that is, $\ell' \to -\infty$. For the normal eye, Equation 4.12 then says that for the eyepiece we have

$$(\mathrm{MP})_e = \frac{-d_0}{f_e'} = \frac{250\,\mathrm{mm}}{f_e'} \tag{4.19}$$

To move the final image to $-\infty$, the real image y_1' must move to the focal point F_e, which is brought about by moving the object y slightly to the left of where it was before, as shown in Figure 4.13; or the object can remain fixed, and the eyepiece moved to the left (see Problem 4.10).

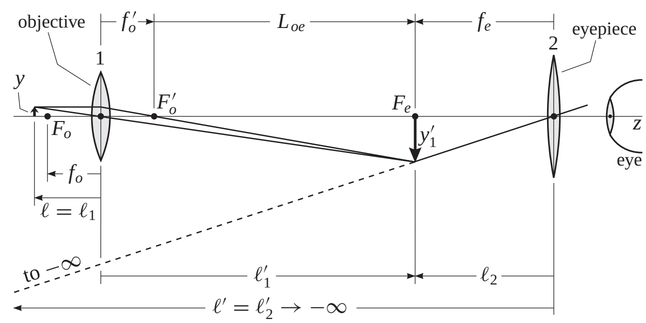

Figure 4.13

Finally, we want to calculate the transverse magnification $(m_T)_o$ of the objective lens. This calculation we do by writing the thin lens equations for the objective from Equations 1.163 and 1.164 as

$$-\frac{1}{\ell} + \frac{1}{\ell_1'} = \frac{1}{f_o'} \tag{4.20}$$

and

$$(m_T)_o = \frac{y_1'}{y} = \frac{\ell_1'}{\ell} \tag{4.21}$$

We are allowed to write these equations for the objective lens in terms of the more convenient ℓ, ℓ' language rather than s, s' language by Equation 1.162; that is, for a thin lens, they are the same. Our next step is to multiply Equation 4.20 through by ℓ_1', and then to solve for ℓ_1'/ℓ: we obtain

$$\frac{\ell_1'}{\ell} = \frac{f_o' - \ell_1'}{f_o'} \tag{4.22}$$

We substitute Equation 4.22 into Equation 4.21 to obtain for the objective lens

$$(m_T)_o = \frac{\ell_1'}{\ell} = \frac{f_o' - \ell_1'}{f_o'} = -\frac{L_{oe}}{f_o'} \tag{4.23}$$

where to obtain the last step we have read by inspection of Figure 4.13 that $\ell_1' = f_o' + L_{oe}$.

Now we want to describe the compound microscope by the system approach, as illustrated by the first part of Example 1.7.1. By inspection of Figure 4.12, we have

f'	n	t
	1	
f_o'		
	1	$f_o' + L_{oe} + f_e'$
f_e'		
	1	

Next, we calculate the system matrix S_{21} for the thin-lens compound microscope

$$
\begin{aligned}
S_{21} &= Z_2 T_{21} Z_1 \\
&= \begin{pmatrix} 1 & -1/f'_e \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ f'_o + L_{oe} + f'_e & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/f'_o \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} -\dfrac{f'_o + L_{oe}}{f'_e} & \dfrac{L_{oe}}{f'_o\, f'_e} \\ f'_o + L_{oe} + f'_e & -\dfrac{L_{oe} + f'_e}{f'_o} \end{pmatrix} = \begin{pmatrix} b & -a \\ -d & c \end{pmatrix}
\end{aligned}
$$

and read off the Gaussian constants:

$$
\left.
\begin{aligned}
a &= -\frac{L_{oe}}{f'_o\, f'_e} & \qquad b &= -\frac{f'_o + L_{oe}}{f'_e} \\
c &= -\frac{L_{oe} + f'_e}{f'_o} & \qquad d &= -\left(f'_o + L_{oe} + f'_e\right)
\end{aligned}
\right\} \tag{4.24}
$$

Using Equations 4.24, because $a = 1/f'$, we have

$$
f' = \frac{1}{a} = -\frac{f'_o\, f'_e}{L_{oe}} \tag{4.25}
$$

and with Equation 4.10

$$
\begin{aligned}
\text{MP} &= \left(\frac{c}{\ell'} - \frac{1}{f'}\right) \frac{d_0}{1 + L_s/\ell'} \\
&= \left(\frac{L_{oe}}{f'_o\, f'_e} - \frac{L_{oe} + f'_e}{f'_o\, \ell'}\right) \frac{d_0}{1 + L_s/\ell'}
\end{aligned} \tag{4.26}
$$

where we have interchanged terms inside the parentheses in the last step.

When we move the final image to $-\infty$ by allowing $\ell' \to -\infty$ in Equation 4.26, we obtain

$$
\begin{aligned}
\text{MP}_{\ell' \to -\infty} &= \left(\frac{L_{oe}}{f'_o\, f'_e}\right) d_0 \\
&= \left(-\frac{L_{oe}}{f'_o}\right) \left(\frac{-d_0}{f'_e}\right) && (4.27\text{a}) \\
&= (m_T)_o\, (\text{MP})_e && (4.27\text{b})
\end{aligned}
$$

where we have used Equations 4.19 and 4.23 in the last step. Thus, we have a very simple interpretation of the MP for a compound microscope when the final image is at $-\infty$; namely, the transverse magnification of the objective times the magnifying power of the eyepiece. Values for these quantities are usually etched on the respective housings of the objective and the eyepiece, and are multiplied together to get the final MP. We also clearly see that small values of f'_o and f'_e give large values for MP.

We can orient Equation 4.27a towards numerics by replacing d_0 with -250 mm for the normal eye, and setting L_{oe} equal to the industry standard of 160 mm:

$$
\text{MP}_{\ell' \to -\infty} = \left(-\frac{160\,\text{mm}}{f'_o}\right) \left(\frac{250\,\text{mm}}{f'_e}\right) \tag{4.28}
$$

We now turn our attention to an expression for ℓ; that is, given ℓ', finite or infinite, we want to calculate ℓ. Substituting the needed expressions from Equations 4.24 and 4.25 into Equation 4.11b, we obtain after some rearranging:

$$
\begin{aligned}
\ell &= \frac{d - b\ell'}{-c + \ell'/f'} \\
&= -f'_o\, \frac{f_e'^2 + \left(f'_o + L_{oe}\right)\left(f'_e - \ell'\right)}{f_e'^2 + L_{oe}\left(f'_e - \ell'\right)}
\end{aligned} \tag{4.29}
$$

To see what happens when $\ell' \to -\infty$, we divide the final expression in Equation 4.29 through by ℓ', and then let $\ell' \to -\infty$ to get

$$
\ell_{\ell' \to -\infty} = -f'_o \left(1 + \frac{f'_o}{L_{oe}}\right) \tag{4.30}
$$

Example 4.1.6 A compound microscope.

Suppose a compound microscope has an objective lens of 10 mm focal length and an eyepiece of 40 mm focal length. The optical tube length is 160 mm, and we assume that the eye is held close enough to the eyepiece that $L_s = 0$. If the final image has $\ell' = -250$ mm, we want to calculate MP for the normal eye. We also would like to determine the distance ℓ of the object from the objective.

We see that $f'_o = 10$ mm, $f'_e = 40$ mm, $L_{oe} = 160$ mm, and $d_0 = -250$ mm. Substituting these values and the given values for L_s and ℓ' into Equation 4.26, we obtain

$$
\begin{aligned}
\text{MP} &= \left(\frac{L_{oe}}{f'_o\, f'_e} - \frac{L_{oe} + f'_e}{f'_o\, \ell'}\right) \frac{d_0}{1 + L_s/\ell'} \\
&= \left(\frac{160}{(10)\,(40)} - \frac{160 + 40}{(10)\,(-250)}\right) \frac{-250}{1} \\
&= -120
\end{aligned}
$$

Substituting the same values into Equation 4.29, we calculate

$$
\begin{aligned}
\ell &= -f'_o\, \frac{f_e'^2 + \left(f'_o + L_{oe}\right)\left(f'_e - \ell'\right)}{f_e'^2 + L_{oe}\left(f'_e - \ell'\right)} \\
&= -10\, \frac{40^2 + (10 + 160)\,(40 - (-250))}{40^2 + 160\,(40 - (-250))} \\
&= -10.6\,\text{mm}
\end{aligned}
$$

If the image is moved to $-\infty$, then we use the simpler Equation 4.27a to calculate

$$\mathrm{MP}_{\ell'\to-\infty} = \left(-\frac{L_{oe}}{f_o'}\right)\left(\frac{-d_0}{f_e'}\right)$$
$$= \left(-\frac{160}{10}\right)\left(\frac{250}{40}\right) = -100$$

and Equation 4.30 to obtain

$$\ell_{\ell'\to-\infty} = -f_o'\left(1 + \frac{f_o'}{L_{oe}}\right)$$
$$= -10\left(1 + \frac{10}{160}\right) = -10.6\,\mathrm{mm}$$

Interestingly, when we compare the two values of ℓ that we have obtained, we see that they are the same to three significant figures. If we display the values to three decimal places, then we see a difference:

$$\ell = -10.604\,\mathrm{mm} \qquad \ell_{\ell'\to-\infty} = -10.625\,\mathrm{mm}$$

In other words, when the value of ℓ is changed only slightly, the value of ℓ' changes from -250 mm to $-\infty$.

4.2 Magnifying Distant Objects

4.2.1 Theory

Instruments for magnifying distant objects are called telescopes. The theory for describing their properties is similar to that for magnifying nearby objects (see Section 4.1.2), except the object is far away for both the unaided and aided eye (see Figure 4.14). Equation 4.5 still holds for these diagrams, which we rewrite and renumber for convenience:

$$\mathrm{MP} = \frac{y'_{\mathrm{eye}}}{y_{\mathrm{eye}}} = \frac{t\,\delta'_{\mathrm{eye}}}{t\,\delta_{\mathrm{eye}}} = \frac{\delta'/n'_{\mathrm{eye}}}{\delta/n'_{\mathrm{eye}}} = \frac{\delta'}{\delta} \tag{4.31}$$

When we compare the diagrams in Figure 4.14 with the similar ones in Figure 4.4, we see that there is no near distance d_0 in Figure 4.14(a) as there is in Figure 4.4(a)—of course, the reason for this difference is that the object is always a distant one, far enough away that the rays reaching the unaided eye in (a), or the lens system in (b), are parallel rays with the angle of deviation δ.

To obtain an expression for MP, we must get an equation for δ' in terms of δ. As a first step, we draw Figure 4.15—this diagram shows an incident ray (one of the parallel set) of angle of deviation δ that passes into the lens system at just the correct height y to emerge and pass through the eyelens center with angle of deviation δ' and $y_c' = 0$. In terms of matrices, we have

$$\begin{pmatrix}\delta'\\ 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ -L_s & 1\end{pmatrix}\begin{pmatrix}b & -a\\ -d & c\end{pmatrix}\begin{pmatrix}\delta\\ y\end{pmatrix}$$
$$= \begin{pmatrix} b\delta - ay \\ -(d + bL_s)\delta + (c + aL_s)y\end{pmatrix} \tag{4.32}$$

Setting the second rows equal to each other, we get

$$0 = -(d + bL_s)\delta + (c + aL_s)y$$

and then solving for y gives

$$y = \frac{d + bL_s}{c + aL_s}\,\delta \tag{4.33}$$

which is the value of y the incident ray should have when it strikes the first surface of the lens system having the Gaussian constants a, b, c, d, as shown in Figure 4.15. We now set

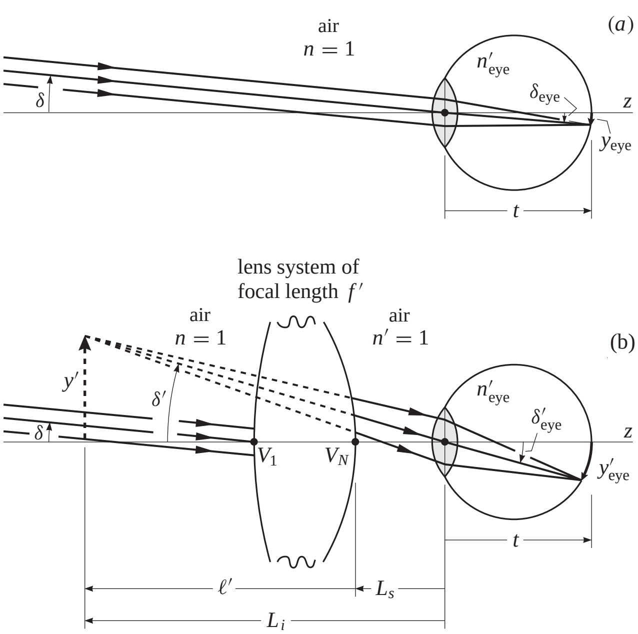

Figure 4.14

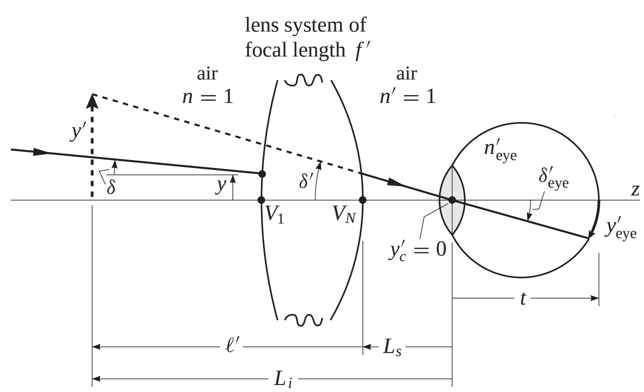

Figure 4.15

the first rows in Equation 4.32 equal to each other, and then substitute the expression for y in Equation 4.33 to get

$$\begin{aligned}\delta' &= b\delta - ay = b\delta - a\,\frac{d + bL_s}{c + aL_s}\,\delta \\ &= \frac{bc - ad}{c + aL_s}\,\delta = \frac{\delta}{c + L_s/f'}\end{aligned} \tag{4.34}$$

where $bc - ad = 1$ (see Equation 1.60), $a = 1/f'$, and f' is the primed focal length of the lens system in Figures 4.14(b) and 4.15. Finally, substituting Equation 4.34 into Equation 4.31, we obtain

$$\text{MP} = \frac{1}{c + L_s/f'} \tag{4.35}$$

As discussed in Case 1 in Section 1.4.2, an afocal or astronomical system is obtained when $a = 0$, which also means $f' = \infty$. Then Equation 4.35 becomes simply

$$\text{MP} = \frac{1}{c} \tag{4.36}$$

and the position of the eye does not matter. However, a telescope can be adjusted to give a finite f' to yield a viewable image at the near point or farther away. Then, we must have $L_s = 0$ in Equation 4.35 to yield Equation 4.36. Usually, a telescope is made so that it is convenient to hold the eye near enough to the eyepiece to take $L_s = 0$.

If we assume $L_s = 0$, then from Figure 4.15 we see that $L_i = \ell'$; we choose to employ ℓ' to give the position of the final image. This value is determined from the properties of the lens system, and we calculate it with the object-image equation, namely Equation 4.11a,

$$\ell' = \frac{d + c\ell}{b + a\ell} = \frac{d + c\ell}{b + \ell/f'}$$

But for a telescope, the object is far enough away that we can write $\ell \to -\infty$; to apply this relation we divide numerator and denominator by ℓ, and then let ℓ go to infinity:

$$\ell' = \frac{d/\ell + c}{b/\ell + a} = \frac{c}{a} = c\,f' \tag{4.37}$$

where $f' = 1/a$.

4.2.2 Telescope: two positive thin lenses

The general system. We draw a simple telescope in air made of two positive, or converging, thin lenses in Figure 4.16. As we show in the diagram, the objective lens

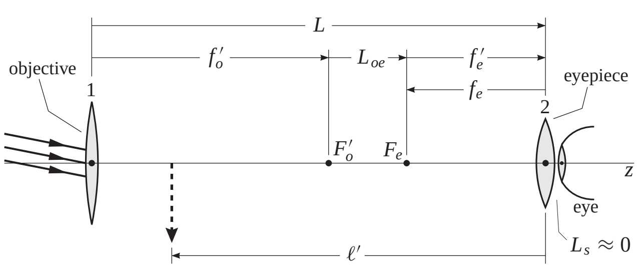

Figure 4.16 The two-lens telescope.

has the larger diameter to increase the amount of light sent through the telescope. We assume that L_s is small enough so that we can take $L_s = 0$; we draw L_{oe} positive to make it easier for analysis; what sign and value it should have will emerge as we analyze this system.

To help construct the matrices for determination of the system matrix, we list the properties of this lens system in the usual tabular form:

f'	n	t
	1	
f'_o		
	1	$f'_o + L_{oe} + f'_e$
f'_e		
	1	

Then we calculate the system matrix S_{21} for the thin-lens telescope

$$\begin{aligned}S_{21} &= Z_2 T_{21} Z_1 \\ &= \begin{pmatrix} 1 & -1/f'_e \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ f'_o + L_{oe} + f'_e & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/f'_o \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -\dfrac{f'_o + L_{oe}}{f'_e} & \dfrac{L_{oe}}{f'_o\,f'_e} \\ f'_o + L_{oe} + f'_e & -\dfrac{L_{oe} + f'_e}{f'_o} \end{pmatrix} = \begin{pmatrix} b & -a \\ -d & c \end{pmatrix}\end{aligned}$$

and read off the Gaussian constants:

$$\left.\begin{aligned} a &= -\frac{L_{oe}}{f'_o\,f'_e} & \qquad b &= -\frac{f'_o + L_{oe}}{f'_e} \\ c &= -\frac{L_{oe} + f'_e}{f'_o} & \qquad d &= -\left(f'_o + L_{oe} + f'_e\right) \end{aligned}\right\} \tag{4.38}$$

Symbolically, we observe that the expressions are the same as for the thin-lens compound microscope (see Equation 4.24).

Because we assume that $L_s = 0$, we substitute the c expression in Equation 4.38 into Equation 4.36 to obtain

$$\text{MP} = \frac{1}{c} = -\frac{f'_o}{L_{oe} + f'_e} \tag{4.39}$$

and then the a and c expressions into Equation 4.37, to get

$$\ell' = \frac{c}{a} = f'_e\left(1 + \frac{f'_e}{L_{oe}}\right) \tag{4.40}$$

We note that ℓ' is independent of f'_o; that is, once f'_e is chosen, then ℓ' is determined only by L_{oe}. When we desire a given ℓ', we solve Equation 4.40 for L_{oe}, and find

$$L_{oe} = -\frac{f_e'^2}{f'_e - \ell'} \tag{4.41}$$

The telescope is normally used to give a virtual image that is viewed by the eye; therefore, ℓ' has a negative value ranging from d_0 (which equals -250 mm for the normal eye) to $-\infty$.

Since $f'_e > 0$ for the telescope under consideration, the denominator in Equation 4.41 is negative; therefore, L_{oe} is negative or zero. When L_{oe} is zero is of special interest, because it leads to the astronomical telescope.

The astronomical telescope. As we mentioned under Equation 4.35, an afocal or astronomical telescope has the property that $a = 0$. Therefore, according to Equation 4.38, we must have $L_{oe} = 0$; that is, the focal points F'_o and F_e will coincide in Figure 4.16. When we substitute $L_{oe} = 0$ into Equation 4.39, we obtain the following simple equation for the magnifying power:

$$\text{MP} = -\frac{f'_o}{f'_e} \tag{4.42}$$

which is the equation most often found in texts. Because MP < 0, the image is inverted. Also, a large MP (in absolute value) is obtained for a large f'_o and a small f'_e.

Example 4.2.1 A telescope with two positive lenses. Referring to Figure 4.16, suppose that $f'_o = 250$ mm and $f'_e = 50$ mm. We assume the lenses are thin. We first want to calculate the magnifying power MP for the system as an astronomical telescope. Second, assuming that $L_s = 0$, we wish to determine L_{oe} and the MP for an image positioned at the near point at a distance of $\ell' = -250$ mm.

For the first case, we use Equation 4.42 to obtain

$$\text{MP} = -\frac{f'_o}{f'_e} = -\frac{250}{50} = -5$$

and draw the ray diagram in Figure 4.17. Since the object is at $-\infty$, the intermediate image—real and inverted—is located on the F'_o focal plane of the objective lens; the ray drawn through the center of the lens determines where the arrowhead is located. Since this image is now the object for the eyepiece, and it is located on the F_e focal plane, an imaginary ray drawn through the arrowhead and the center of the eyepiece determines the parallel direction of the final image rays—the solid lines show the path the rays take out of the lens to the eye, and the dashed lines drawn backward show the virtual image the eye perceives.

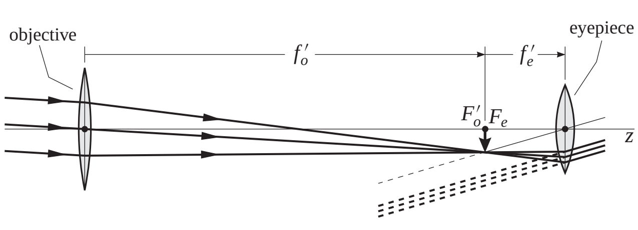

Figure 4.17

For the second case, to create the final image -250 mm to the left of the eyepiece, we move the eyepiece to the left a distance L_{oe}, which means the F_e point moves to the left also (see Figure 4.18). To find the distance L_{oe}, we substitute $f'_e = 50$ mm and $\ell' = -250$ mm into Equation 4.41; we get

$$L_{oe} = -\frac{f_e'^2}{f'_e - \ell'} = -\frac{50^2}{50 + 250} = -8.33\,\text{mm}$$

where the minus sign means the L_{oe} dimension arrow points to the left, in the negative direction, as we show in Figure 4.18. With Equation 4.39, we calculate the magnifying power as

$$\text{MP} = \frac{1}{c} = -\frac{f'_o}{L_{oe} + f'_e} = -\frac{250}{-8.33 + 50} = -6$$

We locate the final image in the diagram of Figure 4.18 by drawing two imaginary rays: one through the arrowhead of the real image and the center of the eyepiece lens, and the other through F_e and the arrowhead. The latter ray emerges from the eyepiece vertical parallel to the symmetry axis (the z axis). The point where the dashed, backward extensions of these two rays intersect locates the arrowhead of the final image. These rays are two of the set used in the three-ray method described in Example 1.5.1 and Figure 1.78.

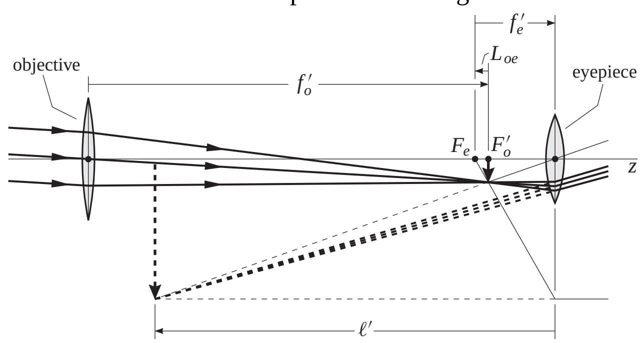

Figure 4.18

We observe that the MP for the final image located at the near point is somewhat larger (in absolute value) than for the case where the final image is at $-\infty$. We also observe that the eyepiece does not have to move to the left very far to bring the image in from $-\infty$ to the near point.

4.2.3 Telescope with a Barlow lens

We have already noted from Equation 4.42 that a larger f'_o makes a larger MP: one way to increase f'_o is to place a diverging lens, called a Barlow lens, to the right of the objective, as we show in Figure 4.19. Viewing the combination of

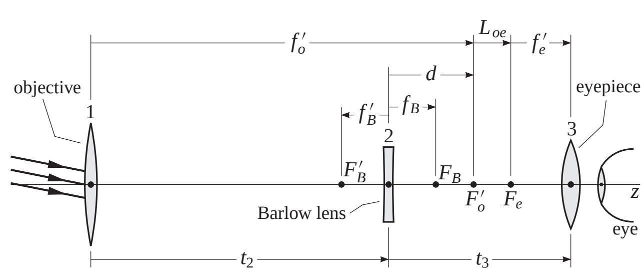

Figure 4.19 The Barlow lens telescope.

the objective and the Barlow lens as a new objective of longer focal length $\bar{f}_o'$, we obtain an astronomical telescope of larger MP. Given the distance d (see Figure 4.19) the Barlow lens is to the left of F_o', we must determine what L_{oe} distance makes this telescope into an afocal or astronomical telescope; that is, what L_{oe} distance makes the Gaussian constant a zero—remember, for an afocal system, $a = 0$.

Our first task is to get an expression for a, which means that we must obtain the system matrix S_{31} for this system. This calculation is somewhat messy to perform by hand, so to reduce the work somewhat we use the distances t_2 and t_3 shown in Figure 4.19 to specify the lens spacing. The telescope is in air, so there are no indices of refraction associated with the t_2 and t_3 distances. We have

$$\begin{aligned} S_{31} &= Z_3 T_{32} Z_2 T_{21} Z_1 \\ &= \begin{pmatrix} 1 & -1/f_e' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ t_3 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/f_B' \\ 0 & 1 \end{pmatrix} \\ &\qquad \begin{pmatrix} 1 & 0 \\ t_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1/f_o' \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} b & -a \\ -d & c \end{pmatrix} \end{aligned} \tag{4.43}$$

After expanding the matrices, we find

$$a = \frac{\left(f_o' - t_2\right)\left(f_e' - t_3\right) + f_B'\left(f_o' + f_e' - t_2 - t_3\right)}{f_o'\, f_e'\, f_B'} \tag{4.44a}$$

$$c = \frac{-\left(f_o' - t_2\right) t_3 + f_B'\left(f_o' - t_2 - t_3\right)}{f_o'\, f_B'} \tag{4.44b}$$

We limit our work to expressions for just a and c because they are the only ones we need for the results we desire. We simplify these expressions by substituting the relations obtained by inspection of Figure 4.19

$$t_2 = f_o' - d \tag{4.45a}$$

$$t_3 = d + L_{oe} + f_e' \tag{4.45b}$$

to get

$$a = -\frac{\left(f_B' + d\right) L_{oe} + d^2}{f_o'\, f_e'\, f_B'} \tag{4.46}$$

$$c = -\frac{d\left(d + L_{oe} + f_e'\right) + f_B'\left(L_{oe} + f_e'\right)}{f_o'\, f_B'} \tag{4.47}$$

To make the system an afocal one, we set $a = 0$ and solve for L_{oe}:

$$L_{oe} = -\frac{d^2}{f_B' + d} = \frac{d^2}{f_B - d} \tag{4.48}$$

where $f_B' = -f_B$ and $f_B > 0$ for a diverging lens so that all the quantities are positive in Equation 4.48. We next simplify c in Equation 4.47 by substituting Equation 4.48 and the relation $f_B' = -f_B$; we obtain after some manipulation

$$c = -\frac{f_e'}{f_o'} \frac{f_B - d}{f_B} \tag{4.49}$$

Then with Equation 4.36, we find the magnifying power for the telescope with the Barlow lens:

$$(\mathrm{MP})_B = \frac{1}{c} = -\frac{f_o'}{f_e'} \frac{f_B}{f_B - d} \tag{4.50}$$

When we compare Equation 4.50 with Equation 4.42, we see that the Barlow lens has increased the magnifying power by a factor

$$\mathrm{M} = \frac{f_B}{f_B - d} \tag{4.51}$$

In practice, $f_B > d > 0$, and how big M can be is limited by lens aberrations.

As we mentioned earlier, it is possible to look at the combination of the objective and Barlow lens as a new objective of focal length $\bar{f}_o'$. To calculate the new focal length, we use the result in Problem 1.38 written in terms of the quantities of this section (see Figure 4.19):

$$\frac{1}{\bar{f}_o'} = \frac{1}{f_o'} + \frac{1}{f_B'} - \frac{t_2}{f_o'\, f_B'}$$

We substitute Equation 4.45a and the relation $f_B' = -f_B$ into the above equation, and then solve for $\bar{f}_o'$:

$$\bar{f}_o' = f_o' \frac{f_B}{f_B - d} \tag{4.52}$$

Substituting Equation 4.51 into Equation 4.52, we see that the new focal length is increased by the factor M:

$$\bar{f}_o' = f_o' \mathrm{M} \tag{4.53}$$

And if we divide this new focal length $\bar{f}_o'$ given in Equation 4.52 by the eyepiece focal length f_e' (as is usual with a two lens astronomical telescope), we get

$$(\mathrm{MP})_B = -\frac{\bar{f}_o'}{f_e'} = -\frac{f_o'}{f_e'} \frac{f_B}{f_B - d} \tag{4.54}$$

which agrees with Equation 4.50.

The only disadvantage of a well-made Barlow lens is a slight loss of light passed through the telescope caused by the rays diverging. An advantage: as we have seen, one way to increase the telescope MP is to use a short focal length f_e', but that requires lens surfaces that are more curved introducing larger aberrations—the Barlow lens allows a return to the larger f_e' for the same or larger MP.

Example 4.2.2 A Barlow lens telescope.
Looking at the diagram in Figure 4.19 for the meaning of the quantities, suppose we assume that the objective has $f'_o = 250\,\text{mm}$, the eyepiece $f'_e = 50\,\text{mm}$, the Barlow lens has the unprimed focal length $f_B = 30\,\text{mm}$, and $d = 10\,\text{mm}$. This telescope is the same one we studied in Example 4.2.1, except that we have added a Barlow lens. Using these given values, we calculate from Equation 4.48 that the spacing between F'_o and F_e is

$$L_{oe} = \frac{d^2}{f_B - d} = \frac{10^2}{30 - 10} = 5\,\text{mm}$$

The M factor is found from Equation 4.51

$$\text{M} = \frac{f_B}{f_B - d} = \frac{30}{30 - 10} = 1.5$$

which means that the MP of the telescope in Example 4.2.1 is increased 1.5 times from -5 to -7.5. Or we can use Equation 4.50 to get the same value:

$$(\text{MP})_B = -\frac{f'_o}{f'_e}\frac{f_B}{f_B - d} = -\frac{250}{50}\frac{30}{30 - 10} = -7.5$$

Using these values, we draw the diagram for this telescope in Figure 4.20. We note how the diverging lens (the Barlow lens) causes the rays through it to diverge from the symmetry axis (the z axis) to make a larger image. This real image is in the F_e focal plane of the eyepiece for imaging at $-\infty$. We have omitted drawing any of the rays from the three-ray method for locating images to reduce clutter in the diagram. We used the matrix ray method described in Chapter 1 to trace the rays. We do not show the eye for lack of space.

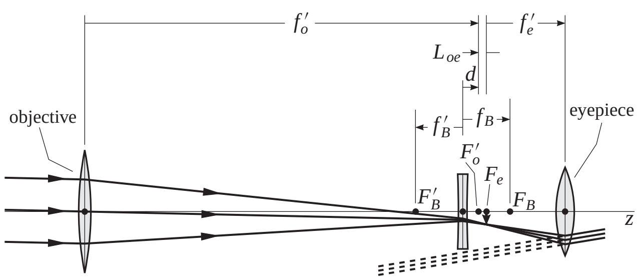

Figure 4.20

4.2.4 The Galilean telescope

A problem with the telescopes we have studied so far is that the final image is inverted. For astronomical work this property is usually not important, but for terrestrial work it is. The basic formula for magnifying power is

$$\text{MP} = -\frac{f'_o}{f'_e}$$

which suggests that making either $f'_o < 0$ or $f'_e < 0$ would produce an erect image. But a negative primed focal length means a diverging, or negative, lens. A diverging lens for the objective is not practical because it reduces the amount of light throughput; that is, the amount of light that passes through the telescope is reduced. However, a diverging lens for the eyepiece is feasible; in fact, Galileo made such a telescope centuries ago. We draw a thin-lens version of a Galilean telescope in Figure 4.21. Comparing this diagram with the one in Figure 4.16, we see that the distance L_{oe} looks different because it must run from F'_o to F_e, and because the eyepiece is now a diverging lens, the F_e and F'_e points are interchanged. In fact, when we work the example that follows, we shall find that F'_o moves to the F_e vicinity.

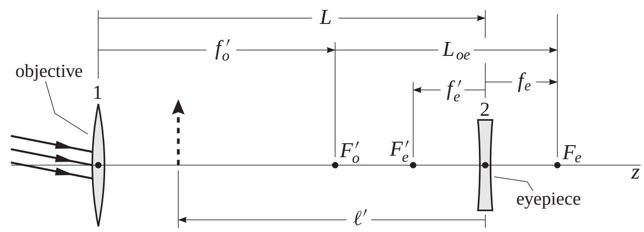

Figure 4.21 The Galilean telescope.

The equations we developed in Section 4.2.2 all work for this telescope, since it is still a two-lens telescope; the only change is that $f'_e < 0$. For a Galilean astronomical telescope, we still have from Equation 4.42

$$\text{MP} = -\frac{f'_o}{f'_e} \tag{4.55}$$

For a telescope that produces an image a finite distance ℓ' from the eyepiece, Equations 4.39 and 4.41 yield

$$\text{MP} = -\frac{f'_o}{L_{oe} + f'_e} \tag{4.56}$$

where

$$L_{oe} = -\frac{f'^2_e}{f'_e - \ell'} \tag{4.57}$$

We have renumbered these equations for convenience of use.

Example 4.2.3 A Galilean telescope.
The Galilean telescope we shall study has an objective lens of $f'_o = 500\,\text{mm}$ and an eyepiece lens of $f'_e = -100\,\text{mm}$. When adjusted to work as an astronomical telescope with ℓ' at $-\infty$, we see that Equation 4.57 predicts that $L_{oe} = 0$. Equation 4.55 says that the magnifying power of this astronomical telescope is

$$\text{MP} = -\frac{f'_o}{f'_e} = -\frac{500}{-100} = 5$$

Because MP is positive, the image is erect. Because L_{oe} is zero, Figure 4.21 indicates that F'_o and F_e must coincide, and we draw the diagram for this telescope in Figure 4.22. We locate the image made by the objective by drawing a ray through the center of the lens to the F_o' plane (remember, this image must be located in the F_o' plane because the rays passing through the objective are a parallel set). This image then becomes the virtual object for the eyepiece; an imaginary ray through the eyepiece and the virtual object locate the direction of the rays going to infinity.

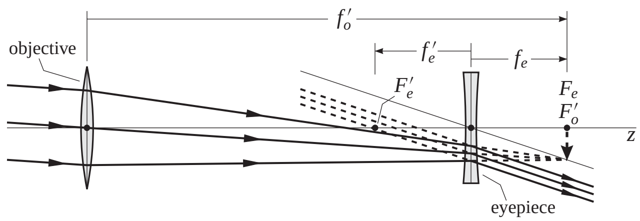

Figure 4.22

Now suppose we wish to bring the final image to the near point; that is, $\ell' = -250$ mm. From Equation 4.57, we calculate how we must move the eyepiece:

$$L_{oe} = -\frac{f_e'^2}{f_e' - \ell'} = \frac{(-100)^2}{-100 + 250} = -66.7\,\text{mm}$$

Then Equation 4.56 gives the magnifying power as

$$\text{MP} = -\frac{f_o'}{f_e' + L_{oe}} = -\frac{500}{-100 + (-66.7)} = 3$$

We observe that the MP is still positive, but is smaller than when the final image was at $-\infty$, which is unlike the behavior of the telescope with two positive lenses (see Example 4.2.1). We diagram this situation for the Galilean telescope in Figure 4.23. Note that we locate the final image with two of the rays from the three-ray method (see Figure 1.81).

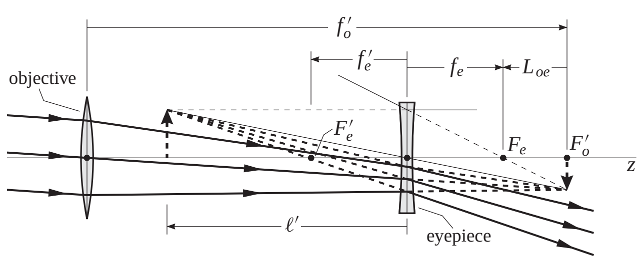

Figure 4.23

Unfortunately, the field of view of a Galilean telescope is smaller than those formed by other telescopes. But, it sometimes still finds use in low-power opera glasses. The fact that no real image is formed by the objective means that there is no place to put crosshairs that can be viewed by the eyepiece. However, this property is useful for a laser beam expander; especially when the laser is a powerful one for there is no place the rays come together to form a real image.

4.2.5 The terrestrial telescope

Another way to create an erect image is to insert an erecting lens between the objective and the eyepiece, an optical system called a terrestrial telescope. A thin-lens version of this system is shown in Figure 4.24. All the lenses are converging (or positive). This telescope is more versatile than the Galilean one, but it does have the disadvantage of a longer mechanical tube length L.

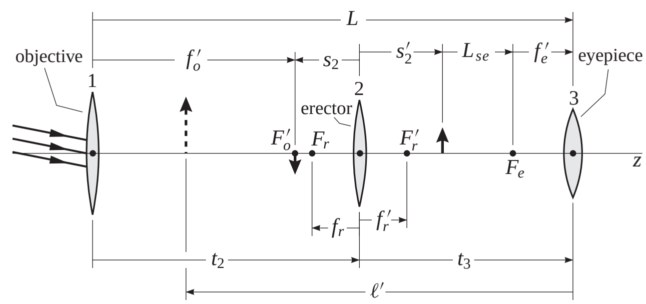

Figure 4.24 The terrestrial telescope.

To simplify the derivation of the equations that govern this system, we mark the distances between the lenses as t_2, t_3, respectively. Then, symbolically, we have the same system as the telescope with a Barlow lens, shown in Figure 4.19, and we can use the same equations we derived for a and c in Equations 4.44. The only difference is the focal lengths of the erecting lens are called f_r, f_r', not f_B, f_B'. Thus, we have

$$a = \frac{\left(f_o' - t_2\right)\left(f_e' - t_3\right) + f_r'\left(f_o' + f_e' - t_2 - t_3\right)}{f_o'\, f_e'\, f_r'} \quad (4.58a)$$

$$c = \frac{-\left(f_o' - t_2\right) t_3 + f_r'\left(f_o' - t_2 - t_3\right)}{f_o'\, f_r'} \quad (4.58b)$$

Now that we have the equations for a and c, we proceed with the diagram for the terrestrial telescope in Figure 4.24. By inspection we read

$$t_2 = f_o' - s_2 \quad (4.59a)$$

$$t_3 = s_2' + L_{se} + f_e' \quad (4.59b)$$

and substitute these equations into Equations 4.58 to get

$$a = \frac{f_r' s_2 - \left(f_r' + s_2\right)\left(s_2' + L_{se}\right)}{f_o' f_e' f_r'} \quad (4.60a)$$

$$c = \frac{f_r' s_2 - \left(f_r' + s_2\right)\left(s_2' + L_{se} + f_e'\right)}{f_o' f_r'} \quad (4.60b)$$

In terms of the quantities relative to the erector lens, we use Equation 1.163 to write

$$-\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_r'} \quad (4.61)$$

and then solve for f_r':

$$f_r' = \frac{s_2\, s_2'}{s_2 - s_2'} \tag{4.62}$$

Substituting Equation 4.62 into Equations 4.60, we simplify the expressions to obtain shorter equations for a and c:

$$a = -\frac{L_{se}}{f_o'\, f_e'}\,\frac{s_2}{s_2'} \tag{4.63a}$$

$$c = -\frac{f_e' + L_{se}}{f_o'}\,\frac{s_2}{s_2'} \tag{4.63b}$$

For an afocal system, we must have $a = 0$; therefore, Equation 4.63a predicts $L_{se} = 0$ and the magnifying power MP for an astronomical telescope is

$$\text{MP} = \frac{1}{c} = -\frac{f_o'}{f_e'}\,\frac{s_2'}{s_2} \tag{4.64}$$

where we have used Equations 4.36 and 4.63b. This equation for MP is positive because $s_2 < 0$, the other terms are all positive quantities (see Figure 4.24).

For a final image located anywhere between $-\infty$ and the near point, assuming that the eye is close enough to the eyepiece so that $L_s = 0$, we use Equation 4.63b to obtain

$$\text{MP} = \frac{1}{c} = -\frac{f_o'}{f_e' + L_{se}}\,\frac{s_2'}{s_2} \tag{4.65}$$

The location of the final image is given by Equation 4.37 and Equations 4.63:

$$\ell' = \frac{c}{a} = f_e'\left(1 + \frac{f_e'}{L_{se}}\right) \tag{4.66}$$

If ℓ' is the given quantity, then we can solve Equation 4.66 for L_{se} to get

$$L_{se} = -\frac{f_e'^2}{f_e' - \ell'} \tag{4.67}$$

We note that symbolically these last two equations have the same form as Equations 4.40 and 4.41 for the two-lens telescope in Section 4.2.2.

Example 4.2.4 A terrestrial telescope.
We obtain the afocal (or astronomical) terrestrial telescope shown in Figure 4.25 when we choose $f_o' = 110$ mm for the objective, and $f_e' = 44$ mm for the eyepiece. For the erector, we select $s_2 = -41$ mm and $s_2' = 82$ mm which gives $f_r' = 27.3$ mm from Equation 4.62.

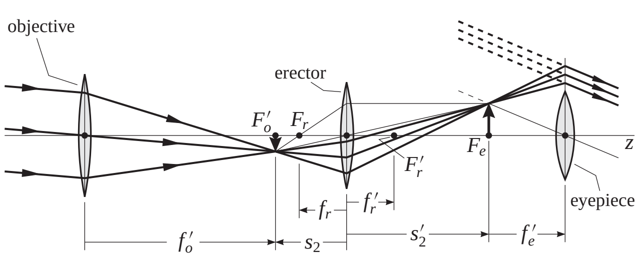

Figure 4.25

Used as an astronomical telescope, Equation 4.64 gives a positive magnifying power for an erect image:

$$\text{MP} = -\frac{f_o'}{f_e'}\,\frac{s_2'}{s_2} = -\frac{110}{44}\,\frac{82}{-41} = 5$$

By inspection of Figure 4.24, we can obtain the expression for the mechanical tube length L of the telescope,

$$L = f_o' - s_2 + s_2' + L_{se} + f_e' \tag{4.68}$$

and then substitute the given values to get the numerical value:

$$L = 110 - (-41) + 82 + 0 + 44 = 277\,\text{mm}$$

where, remember, $L_{se} = 0$ for an astronomical telescope.

If we want to use the telescope to create a final image a finite distance away, say $\ell' = -250$ mm (we assume the eye is close enough to the eyepiece so that $L_s = 0$), then we must use Equation 4.67 to calculate L_{se}:

$$L_{se} = -\frac{f_e'^2}{f_e' - \ell'} = \frac{44^2}{44 + 250} = -6.59\,\text{mm}$$

which means the eyepiece must be pushed to the left of its position in Figure 4.25 to give the diagram in Figure 4.26. The magnifying power of this telescope is obtained from Equation 4.65:

$$\text{MP} = -\frac{f_o'}{f_e' + L_{se}}\,\frac{s_2'}{s_2} = -\frac{110}{44 - 6.59}\,\frac{82}{-41} = 5.88$$

again a positive value, but a little larger than before. We use Equation 4.68 to get the mechanical tube length:

$$L = 110 - (-41) + 82 - 6.59 + 44 = 270\,\text{mm}$$

Note that in both Figures 4.25 and 4.26 we have drawn one or two rays (sometimes imaginary) from the three-ray method to locate the images (see Figure 1.78).

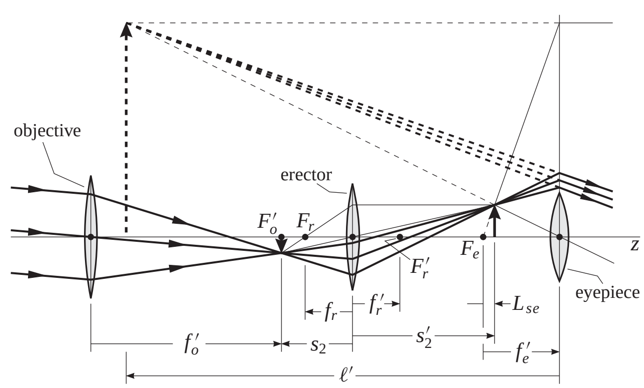

Figure 4.26

Problems

Note: Assume all the lenses are thin.

4.1 A converging lens with a focal length of 10.0 mm is used as a simple magnifier. Suppose the lens is held close enough to the eye to make $L_s = 0$ and positioned to form a virtual image at the normal near point, a distance of 250 mm from the lens. (a) Calculate the magnifying power MP. (b) Calculate the distance ℓ the object is from the lens.

4.2 Suppose the lens in Problem 4.1 is moved to form the virtual image at $-\infty$. Calculate (a) MP, and (b) ℓ.

4.3 Derive Equation 4.16.

4.4 A reading glass has a focal length of 125 mm. It is held 100 mm from a page in a book, and 300 mm from the eye. (a) For a normal near point, determine MP using Equation 4.16. (b) Calculate the distance of the image from the lens.

4.5 We wish to investigate the reading glass situation differently than we did with Equation 4.16; that is, we want to keep L_s constant rather than L_o, and obtain MP as a function of ℓ. (a) Start with either of the two expressions given for MP in Equation 4.10, and write MP for a thin lens as a function of ℓ regarding f', L_s, d_0 as fixed or given quantities. (b) With the same values for f', L_s, d_0 that you used in Problem 4.4, graph MP versus ℓ from $\ell = 0$ to -125 mm. (c) Why do you stop the graph at $\ell = -125$ mm? (d) What is the maximum value of MP, and where is the object when it occurs? (e) Where is the virtual image when MP is maximum? (f) Does the maximum value for MP agree with the result given by Equation 4.12?

4.6 A craftsman with normal vision uses a small lens of magnifying power 25 to view an electronic solder joint; the lens is positioned to place the image at $-\infty$. (a) Determine the focal length of the lens? (b) What is the object distance?

Note: In the following microscope problems, assume a normal eye ($d_0 = -250$ mm) and that the eye is close enough to the eyepiece to make $L_s = 0$—unless explicitly stated otherwise in the problem.

4.7 In a compound microscope, the objective and eyepiece have focal lengths of 10.0 mm and 50.0 mm, respectively. The optical tube length is 140 mm. (a) Calculate MP when the image is 250 mm from the eyepiece. (b) Determine the distance the object is from the objective lens. (c) If the eyepiece is adjusted to move the image to $-\infty$, determine MP. (d) What is the mechanical tube length?

4.8 The objective of a compound microscope has a focal length of 20.0 mm and the eyepiece has a focal length of 10.0 mm. For a mechanical tube length of 100 mm, and the image at $-\infty$, calculate (a) the magnifying power MP, and (b) the object distance.

4.9 A compound microscope consist of an objective of 20.0 mm focal length and an eyepiece of 50.0 mm focal length. The optical tube length is the standard 160 mm. Calculate (a) the MP, and (b) the object distance ℓ when the final image is at $-\infty$.

4.10 (a) Show how to solve Equation 4.29 for L_{oe} to obtain

$$L_{oe} = -\frac{f_o'^2}{f_o' + \ell} - \frac{f_e'^2}{f_e' - \ell'} \tag{4.69}$$

For a given telescope, this equation gives L_{oe} as a function of ℓ, ℓ'. (b) Show that when $\ell' \to -\infty$, Equation 4.69 becomes

$$(L_{oe})_{\ell' \to -\infty} = -\frac{f_o'^2}{f_o' + \ell} \tag{4.70}$$

where now ℓ has the appropriate value to correspond to $\ell' \to -\infty$. The quantity $(L_{oe})_{\ell' \to -\infty}$ is the distance from F_o' to the intermediate image y_1' (see Figure 4.27); it is also the distance from F_o' to the F_e point when the final image is at $-\infty$. As illustrated in Figure 4.27, to place the image at a position other than $-\infty$ we move the eyepiece to the left a distance ΔL_{oe}, where by inspection of Equation 4.69 and Figure 4.27, we see that

$$\Delta L_{oe} = -\frac{f_e'^2}{f_e' - \ell'} \tag{4.71}$$

and we can rewrite Equation 4.69 as

$$L_{oe} = (L_{oe})_{\ell' \to -\infty} + \Delta L_{oe} \tag{4.72}$$

(c) Except for some minor notational differences, show that when Equation 4.30 is solved for L_{oe}, the result agrees with Equation 4.70.

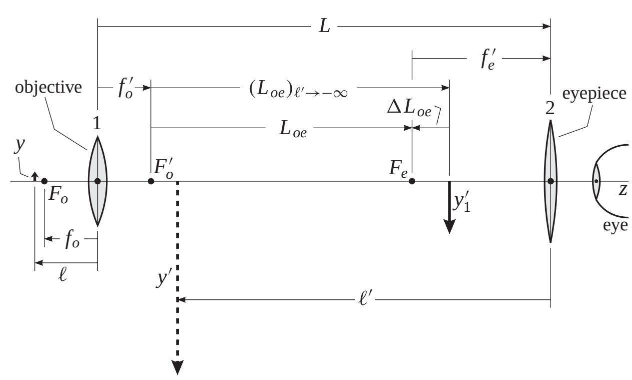

Figure 4.27

4.11 Suppose the microscope is the same as the one described in Problem 4.9. (a) Use Equation 4.71 to calculate how far the eyepiece must move to the left to place the final image at $\ell' = -250\,\text{mm}$. (b) Calculate $(L_{oe})_{\ell' \to -\infty}$ with Equation 4.70. Does it agree with the L_{oe} value in Problem 4.9? (c) Calculate L_{oe} using Equation 4.72 (which, of course, agrees with the result you would get from Equation 4.69).

4.12 You have two lenses on hand, one of 20.0 mm focal length, the other of 40.0 mm focal length. You wish to make a compound microscope with the first lens as the objective, the second as the eyepiece. (a) If you place the object 23.0 mm from the objective and adjust the eyepiece for a final image at $-\infty$, calculate the optical tube length, the mechanical tube length, and the magnifying power. (b) How far must you move the eyepiece to put the final image at $\ell' = -250\,\text{mm}$? Determine the new values for the optical tube length, the mechanical tube length, and the magnifying power.

4.13 A compound microscope has an objective and eyepiece of 3.50 mm and 12.0 mm focal lengths, respectively. The objective forms a real, inverted image 160 mm beyond its F_o' focal point. What is the magnifying power of this microscope, if the final image is formed at $-\infty$?

4.14 For a compound microscope, the distance between an 8.00 mm objective and a 6.00 mm eyepiece is 190 mm. If the final image is located at $-\infty$, calculate (a) the transverse magnification of the objective, (b) the object distance, and (c) the magnifying power.

4.15 For the same compound microscope and object distance in Problem 4.14, suppose the eyepiece is moved to place the final image at -250 mm. Find the magnifying power.

4.16 The optical tube length of a compound microscope is 160 mm. The focal lengths of the objective and eyepiece are 10.0 mm and 20.0 mm, respectively. (a) Determine the focal length f' of the system. (b) If the final image is at $-\infty$, find the object distance ℓ, and magnifying power MP.

4.17 The focal lengths of the objective and eyepiece of a compound microscope are 12.0 mm and 23.0 mm, respectively; the optical tube length is 160 mm. The final image is viewed at the near point, which for this observer is 300 mm from the eyes. Find (a) the distance the object is from the objective, and (b) the magnifying power MP.

4.18 Perform the matrix multiplication work to obtain the Gaussian constants in Equation 4.24.

4.19 Derive Equation 4.29.

4.20 Show the steps for obtaining Equation 4.30 from Equation 4.29.

Note: In the following telescope problems, assume a normal eye ($d_0 = -250\,\text{mm}$) and that the eye is close enough to the eyepiece to make $L_s = 0$—unless explicitly stated otherwise in the problem. Also, when the telescope is referred to as an astronomical telescope, we mean one for which $a = 0$; that is, an afocal system.

4.21 Perform the matrix multiplication in S_{21} to obtain the Gaussian constant expressions of Equation 4.38.

4.22 An astronomical telescope has an objective and eyepiece of focal lengths 3050 mm and 25.0 mm, respectively. Determine the magnifying power MP.

4.23 For an astronomical telescope, the eyepiece has a focal length of 30.0 mm and a magnifying power of -25.0. Calculate the focal length of the objective.

4.24 It is determined that an astronomical telescope has an magnifying power of -22.0 and a mechanical tube length of 300 mm. Find the focal lengths of the objective and the eyepiece.

4.25 The objective and eyepiece lenses have focal lengths of 900 mm and 40.0 mm, respectively. (a) When used as an astronomical telescope, determine the magnifying power. (b) The eyepiece is adjusted so that the image is viewed at the normal near point. Calculate the magnifying power.

4.26 A distant object in the shape of a rod subtends an angle of $\delta = -0.0100$ rad at the objective of a telescope (the shape of the object is not important, it is chosen as a rod only for convenience of thinking). The objective has a focal length of 852 mm and an eyepiece of focal length 30.0 mm. The eyepiece of the telescope is adjusted to give a real image on a screen placed 250 mm from the eyepiece. Find (a) the optical tube length L_{oe}, (b) the mechanical tube length L, (c) the magnifying power, (d) the emergent angle of inclination δ' of the ray that emerges from the center of the eyepiece, (e) the height of the image on the screen, and (f) the height of the image formed by the objective. (g) Draw a rough ray diagram of this optical system. *Hint:* Remember that the basic definition of magnifying power is δ'/δ.

4.27 The diameter of the moon is 3480 km and the earth-moon distance is about 380, 000 km. (a) Ignoring the radii of the earth and moon, what angle is subtended by the moon at the earth? Suppose the astronomical telescope of Problem 4.22 is used to look at the moon. (b) What angle is subtended by the image in the telescope? (c) Determine the diameter of the real image created by the objective.

4.28 Obtain Equations 4.44 by multiplying the matrices in Equation 4.43.

4.29 Show how to get Equations 4.46 and 4.47.

4.30 Write down the steps to obtain Equation 4.48.

4.31 Show the details for deriving Equation 4.50.

4.32 Suppose a telescope with an objective and eyepiece of 500 mm and 50.0 mm focal lengths, respectively, is fitted with a Barlow lens of 20.0 mm focal length. The Barlow lens is moved to the left of F'_o to made $d = 8.00$ mm. If the telescope is used as an afocal system, (a) calculate L_{oe}, MP, M, $(\text{MP})_B$, $\bar{f}'_o$. (b) Draw a rough diagram that shows the positions of the lenses, and the positions of the points F'_o, F_e, F_B. Label the distances d, L_{oe}, f_B. Also, show where the real image is located.

4.33 You buy a Barlow lens that has an 80 mm focal length, and is designed to work at $\text{M} = 2$ in an afocal telescope. (a) Calculate the values of d and L_{oe}. (b) Draw a diagram that shows the diverging lens and the points F'_o, F_e, F_B. Label the distances d, L_{oe}, f_B, and then show where the real image is located.

4.34 An 80 mm focal length Barlow lens is placed 60 mm to the left of F'_o of an objective of 1600 mm focal length in an astronomical telescope ($a = 0$). (a) Calculate L_{oe}, M, and $\bar{f}'_o$. (b) Draw a diagram that shows the points F'_o, F_e, F_B, and the diverging lens. Show where the real image is located, and label the distances d, L_{oe}, f_B.

4.35 A Galilean telescope consists of an objective with focal length $f'_o = 250$ mm and an eyepiece with focal length $f'_e = -50.0$ mm. When used as an astronomical telescope, determine (a) the magnifying power, and (b) the mechanical tube length. (c) When used as a two-lens telescope with the eyepiece replaced by a lens of $f'_e = 50.0$ mm (see Example 4.2.1), what is the mechanical tube length?

4.36 The same Galilean telescope is used as in Problem 4.35, but the eyepiece is adjusted to form the final image at $\ell' = -250$ mm. Calculate (a) the optical tube length L_{oe}, (b) the magnifying power, and (c) the mechanical tube length L. (d) Use the results in Example 4.2.1 to determine the mechanical tube length of that telescope, which is the same as the Galilean telescope we are considering except the eyepiece is a converging lens.

4.37 A Galilean telescope having a magnifying power of 3 forms the final image at $-\infty$. The objective has a focal length of $f'_o = 130$ mm. What is the focal length of the eyepiece?

4.38 The magnifying power of a Galilean telescope is 3.5. To place the final image at $-\infty$, the mechanical tube length measures 100 mm. Determine the focal lengths of the objective and the eyepiece.

4.39 The distance between the objective and the eyepiece of a Galilean telescope is 100 mm. The magnifying power is 4.2 and the final image is viewed at the near point. Calculate f'_o, f'_e, and L_{oe}.

4.40 The mechanical tube length and the optical tube length of a Galilean telescope is 100 mm and -4.50 mm, respectively. The magnifying power is 3.5. Find f'_o, f'_e, and ℓ'.

4.41 Start with Equations 4.58 and show the steps to obtain Equations 4.60.

4.42 Obtain the Equations 4.63.

4.43 Show the details to get Equations 4.66 and 4.67.

4.44 A terrestrial telescope has the following lens values: $f'_o = 800$ mm, $f'_r = 180$ mm, and $f'_e = 40.0$ mm. The magnitudes of the object and image distances relative to the erecting lens are the same. If the final image is formed at $-\infty$, (a) find the magnifying power, (b) the object and image distances relative to the erecting lens, and (c) the mechanical tube length (that is, the distance between the objective and the eyepiece).

4.45 The terrestrial telescope has the same properties as in Problem 4.44, but the eyepiece is adjusted to form the final image at the near point. Determine (a) L_{se}, (b) the magnifying power, and (c) the mechanical tube length.

4.46 The objective and erecting lenses of a terrestrial telescope are 800 mm and 200 mm, respectively. The magnifying power of the telescope is given as 20, and the object/image relation relative to the erecting lens is $s'_2 = -2s_2$. Calculate (a) the values of s_2 and s'_2, (b) the focal length of the eyepiece, and (c) the mechanical tube length of the telescope.

4.47 Suppose you would like to make a terrestrial telescope with a mechanical tube length L of 500 mm and a magnifying power of 25. You have on hand a converging lens of 50.0 mm focal length to use as an erecting lens, and you choose that $s'_2 = -s_2$. Find the values of s_2 and s'_2, and the focal lengths of objective and the eyepiece.

Chapter 4: Answers to Problems

4.1 (a) 26; (b) −9.62 mm

4.2 (a) 25; (b) −10 mm

4.4 (a) 1.56; (b) −500 mm

4.5 (a) $\dfrac{d_0\, f'}{L_s\, f' + (L_s + f')\ell}$
(d) 2

4.6 (a) 10 mm; (b) −10 mm

4.7 (a) −89; (b) −10.7 mm; (c) −70; (d) 200 mm

4.8 (a) −87.5; (b) −25.7 mm

4.9 (a) −40; (b) −22.5 mm

4.11 (a) −8.33 mm; (b) 160 mm; (c) 152 mm

4.12 (a) 133 mm, 193 mm, −41.7;
(b) −5.52 mm, 128 mm, 188 mm, −48.3

4.13 -879

4.14 (a) −8.36 mm; (b) −22; (c) −917

4.15 (a) −939

4.16 (a) −1.25 mm; (b) −10.6 mm; −200

4.17 (a) −12.9 mm; (b) −185

4.22 −122

4.23 750 mm

4.24 (a) 287 mm; (b) 13.0 mm

4.25 (a) −22.5; (b) −26.1

4.26 (a) 4.09 mm; (b) 886 mm; (c) -25.0; (d) 0.250 rad;
(e) 62.5 mm; (f) −8.52 mm

4.27 (a) 0.00916 rad = 0.525 deg; (b) 1.12 rad = 64.0 deg;
(c) 27.9 mm

4.32 (a) 5.33 mm, −10, 1.67, −16.7, 833 mm

4.40 (a) 40 mm, 40 mm

4.34 (a) 240 mm, 4, 6400 mm

4.35 (a) 5; (b) 200 mm; (c) 300 mm

4.36 (a) −12.5 mm; (b) 4; (c) 188 mm; (d) 292 mm

4.37 −43.3 mm

4.38 140 mm, −40.0 mm

4.39 131 mm, −27.8 mm, −3.47 mm

4.40 140 mm, −35.5 mm, −316 mm

4.44 (a) 20; (b) −360 mm, 360 mm; (c) 1560 mm

4.45 (a) −5.52 mm; (b) 23.2; (c) 1554 mm

4.46 (a) −300 mm, 600 mm; (b) 80 mm; (c) 1780 mm

4.47 −100 mm, 100 mm, 288 mm, 11.5 mm

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